4.8 Quadrature Amplitude Modulation (QAM)

Definition 4.90. In ***quadrature amplitude modulation*** (***QAM***) or ***quadrature multiplexing***, two baseband real-valued signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the corresponding QAM signal:

$$x_{\mathrm{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

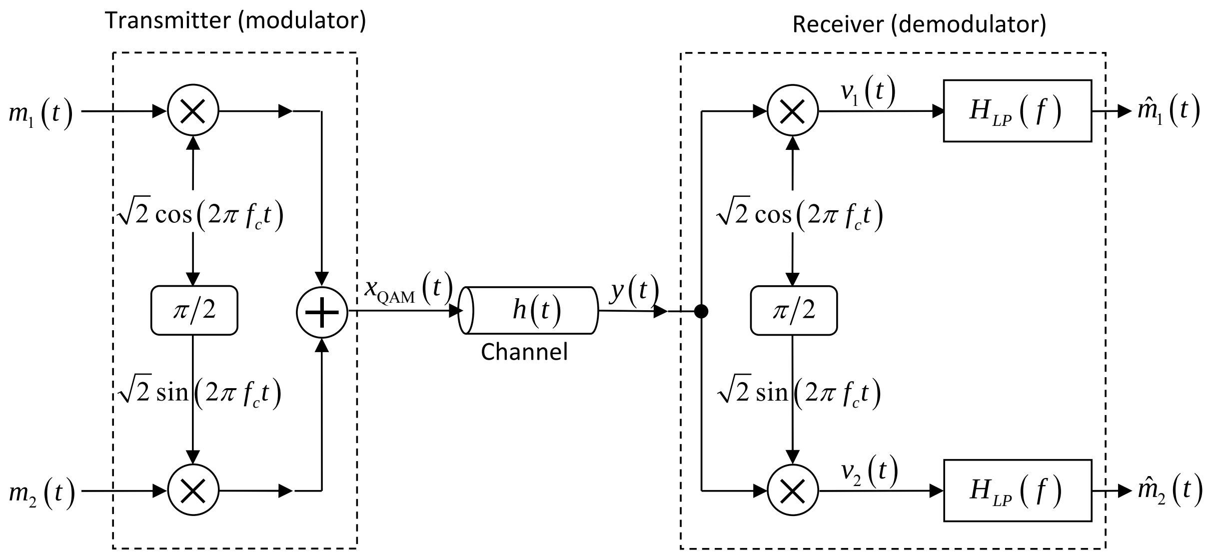


Figure 32: QAM Scheme

- QAM operates by transmitting two DSB signals via carriers of the same frequency but in phase quadrature.
- Both modulated signals simultaneously occupy the same frequency band.
- The "cos" (upper) channel is also known as the ***in-phase*** (***I***) channel and the "sin" (lower) channel is the ***quadrature*** (***Q***) channel.

4.91. ***Demodulation***: Under the usual assumption ($B < f_c$), the two baseband signals can be separated at the receiver by synchronous detection:

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\cos(2\pi f_c t)\right\} = m_1(t) \quad (65)$$

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\sin(2\pi f_c t)\right\} = m_2(t) \quad (66)$$

To see (65), note that

$$
\begin{aligned}
v_1(t) &= x_{\text{QAM}}(t)\sqrt{2}\cos(2\pi f_c t) \\
&= \left(m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t)\right)\sqrt{2}\cos(2\pi f_c t) \\
&= m_1(t)\,2\cos^2(2\pi f_c t) + m_2(t)\,2\sin(2\pi f_c t)\cos(2\pi f_c t) \\
&= m_1(t)\left(1+\cos(2\pi(2f_c)t)\right) + m_2(t)\sin(2\pi(2f_c)t) \\
&= m_1(t) + m_1(t)\cos(2\pi(2f_c)t) + m_2(t)\cos(2\pi(2f_c)t - 90^\circ)
\end{aligned}
$$

- Observe that $m_1(t)$ and $m_2(t)$ can be separately demodulated.

4.92. Suppose, during a time interval, the messages $m_1(t)$ and $m_2(t)$ are constant. Consider the signal $m_1\sqrt{2}\cos(2\pi f_c t) + m_2\sqrt{2}\sin(2\pi f_c t)$.

Example 4.93. $(1)\sqrt{2}\cos(2\pi f_c t) + (1)\sqrt{2}\sin(2\pi f_c t)$

Example 4.94. $3\sqrt{2}\cos(2\pi f_c t) + 4\sqrt{2}\sin(2\pi f_c t)$

4.95. Sinusoidal form (envelope-and-phase description [3, p. 165]):

$$x_{\text{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

where

$$
\begin{aligned}
\text{envelope:} \quad & E(t) = |m_1(t) - jm_2(t)| = \sqrt{m_1^2(t) + m_2^2(t)} \\
\text{phase:} \quad & \phi(t) = \angle\left(m_1(t) - jm_2(t)\right)
\end{aligned}
$$

Example 4.96. In a QAM system, the transmitted signal is of the form

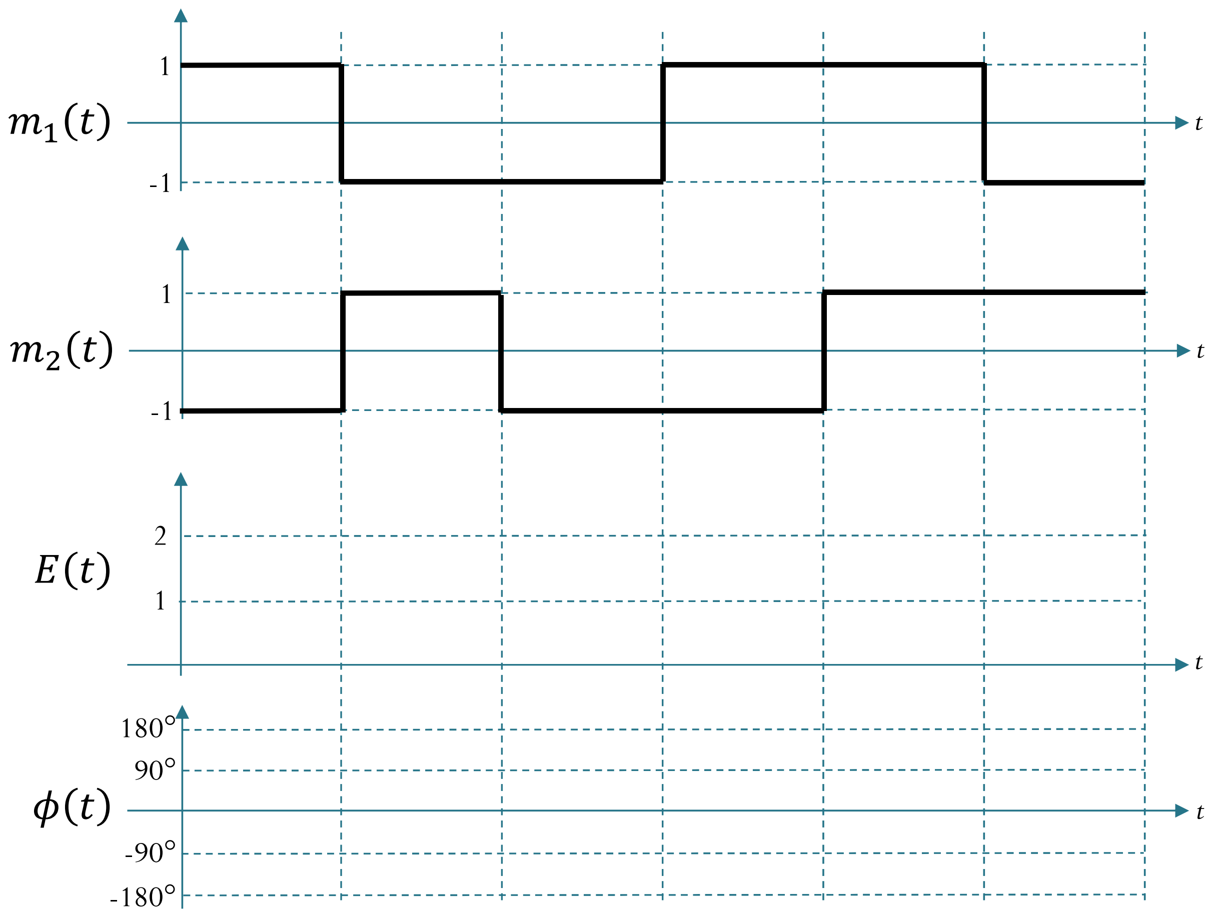
$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

Here, we want to express $x_{\text{QAM}}(t)$ in the form

$$x_{\text{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

where $E(t) \geq 0$ and $\phi(t) \in (-180°, 180°]$.

Consider $m_1(t)$ and $m_2(t)$ plotted in the figure below. Draw the corresponding $E(t)$ and $\phi(t)$.



4.97. $m_1\sqrt{2}\cos(2\pi f_c t) + m_2\sqrt{2}\sin(2\pi f_c t)$

4.98. Complex form of QAM signal:

$$x_{\text{QAM}}(t) = \sqrt{2}\text{Re}\left\{(\mathbf{m}(t))\, e^{j2\pi f_c t}\right\}$$

where[20] $\mathbf{m}(t) = m_1(t) - jm_2(t)$.

- We refer to $\mathbf{m}(t)$ as the ***complex envelope*** (or ***complex baseband signal***) and the signals $m_1(t)$ and $m_2(t)$ are known as the ***in-phase*** and ***quadrature(-phase)*** components of $x_{\text{QAM}}(t)$.
- The term "quadrature component" refers to the fact that it is in phase quadrature ($\pi/2$ out of phase) with respect to the in-phase component.
- Key equation:

$$\text{LPF}\left\{\underbrace{\left(\text{Re}\left\{\mathbf{m}(t) \times \sqrt{2}e^{j2\pi f_c t}\right\}\right)}_{x_{\text{QAM}}(t)} \times \left(\sqrt{2}e^{-j2\pi f_c t}\right)\right\} = m(t).$$

4.99. Three equivalent ways of saying exactly the same thing:

(a) the complex-valued envelope $\mathbf{m}(t)$ complex-modulates the complex carrier $e^{j2\pi f_c t}$,

- So, now you can understand what we mean when we say that a complex-valued signal is transmitted.

(b) the real-valued amplitude $E(t)$ and phase $\phi(t)$ real-modulate the amplitude and phase of the real carrier $\cos(2\pi f_c t)$,

(c) the in-phase signal $m_1(t)$ and quadrature signal $m_2(t)$ real-modulate the real in-phase carrier $\cos(2\pi f_c t)$ and the real quadrature carrier $\sin(2\pi f_c t)$.

[20] If we use $-\sin(2\pi f_c t)$ instead of $\sin(2\pi f_c t)$ for $m_2(t)$ to modulate,

$$\begin{aligned} x_{\text{QAM}}(t) &= m_1(t)\sqrt{2}\cos(2\pi f_c t) - m_2(t)\sqrt{2}\sin(2\pi f_c t) \\ &= \sqrt{2}\,\text{Re}\left\{\mathbf{m}(t)\, e^{j2\pi f_c t}\right\} \end{aligned}$$

where

$$\mathbf{m}(t) = m_1(t) + jm_2(t).$$

Emphasize that there are two messages

$$\text{①}\quad x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t)$$

$$\Leftrightarrow m_1(t)\sqrt{2}\angle 0^\circ + m_1(t)\sqrt{2}\angle -90^\circ$$

$$= E(t)\sqrt{2}\angle\phi(t)$$

$$\text{②}\quad \Leftrightarrow \sqrt{2}E(t)\cos\big(2\pi f_c t + \phi(t)\big)$$

Emphasize that the messages are embedded in both amplitude and phase of the carrier

$e^{jx} = \cos(x) + j\sin(x)$ $\quad$ $-je^{jx} = -j\cos(x) + \sin(x)$

$\cos(x) = \text{Re}\{e^{jx}\}$ $\quad$ $\sin(x) = \text{Re}\{-je^{jx}\}$

$$\text{③}\quad x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\text{Re}\{e^{j2\pi f_c t}\} + m_2(t)\sqrt{2}\text{Re}\{-je^{j2\pi f_c t}\}$$

$$= \sqrt{2}\text{Re}\{\big(m_1(t) - jm_2(t)\big)e^{j2\pi f_c t}\}$$

Emphasize the use of the combined complex-valued representation of the two messages.

Figure 33: Three Forms of QAM

4.100. References: [3, p 164–166, 302–303], [14, Sect. 2.9.4], [5, Sect. 4.4], and [9, Sect. 1.4.1]

4.101. Question: In engineering and applied science, measured signals are real. Why should real measurable effects be represented by complex signals?

Answer: One complex signal (or channel) can carry information about two real signals (or two real channels), and the algebra and geometry of analyzing these two real signals as if they were one complex signal brings economies and insights that would not otherwise emerge. [9, p. 3]